

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that : the sequence $\{F_n(t)\}$ of distribution functions of student's t with n degree of freedom satisfies for every t the relation

$$\lim_{n \rightarrow \infty} F_n(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^t e^{-t^2/2} dt.$$

17. Drive Fisher's z -distribution.
18. Prove that : An unbiased estimates U of the parameters Q is the most efficient if and only if
- the estimate U is sufficient
 - for $g(u, Q) > 0$, the density $g(u, Q)$ almost every where's satisfies the relation.

$$\frac{\partial \log g(u, Q)}{\partial Q} = c(u - Q), \quad C \text{ is independent of } u \text{ (it may depend on } Q).$$
19. Show that : if the point ξ in sample space is a r.v of the continuous type with density $f(\xi, Q)$, where the parameter Q is unknown and $H_0(Q = Q_0)$ and $H_1(Q = Q_1)$ ($Q_0 \neq Q_1$) are the null and alternative hypotheses respectively, then a most powerful test exists.
20. Derive the OC function of the sequential probability Ratio Test.

APRIL/MAY 2025

GEMA24A/DEMA24A — MATHEMATICAL STATISTICS

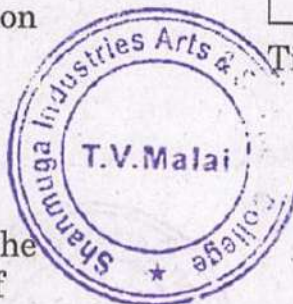
Time : Three hours

Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

- Definition of Random method.
- Define Random sample.
- What is sample space?
- What is statistics?
- Define χ^2 distribution.
- Define statistical hypotheses.
- Define consistent estimates.
- Define sequential probability Ratio Test.



9. Write determination of A and B values.
10. Write the formula of sequential probability Ratio Test.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) The random variable $X_k (k = 1, 2, \dots, 16)$ are, independent and have the same density

$$f(x) = \frac{1}{2\sqrt{2\pi}} \exp\left[-\frac{1}{2} \cdot \frac{(x-1)^2}{4}\right] \text{ to find } \bar{X}.$$

Or

- (b) Derive the χ^2 distribution.
12. (a) The following sample correlation co-efficient were obtained from a simple sample size drawn from a three-dimensional normal population: $r_{12} = 0.2$, $r_{13} = 0.4$, $r_{23} = 0.35$. Then test the hypothesis $H_0 = (P_{12} = P_{13} = P_{23} = 0)$ at the significance level $\alpha = 0.05$.

Or

- (b) Drive χ^2 test.

13. (a) To find the characteristic X of element of a population has the binormal distribution that is

$$P_k = P(X = k) = \binom{r}{k} p^k q^{r-k}$$

($q = 1 - p, 0 < p < 1, k = 0, 1, 2, \dots, r$), where p is an unknown parameter which is to be estimated from a simple sample of n elements drawn from this population.

Or

Let V be an unbiased estimate of the parameter Q , and let U be a sufficient estimates of Q . Then prove that the random variable $E(V/U)$ is an unbiased estimates of Q .

14. (a) Drive One-way classification.

Or

- (b) Drive multiple classification.

15. (a) Prove that : If the variance of z is different from zero, the probability that $n = \infty$ is equal to zero.

Or

- (b) Drive the Determination of A and B .

